① Given $f(x)=\sec(x)$, find the tangent line linearization of $f(x)$ centered at $x=1$. How close to one do you have to stay to $x=1$ to have an error of less than 0.001?

$y=m(x-x_1)+y_1$

$y=f'(a)(x-a)+f(a)$

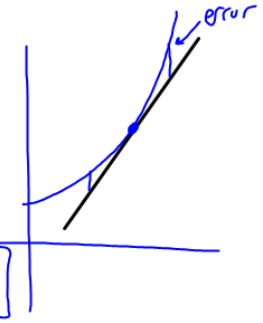
$y=f'(a)(x-1)+\sec(1)$

pt. $(x=1,\ y=\sec(1))$

$f'(x)=\sec x\tan x$

$f'(1)=\sec(1)\tan(1)$

$y=\sec(1)\tan(1)(x-1)+\sec(1)$



$\text{error}=f(x)-L(x)$

$0.001=\sec(x)-\left[\sec(1)\tan(1)(x-1)+\sec(1)\right]$

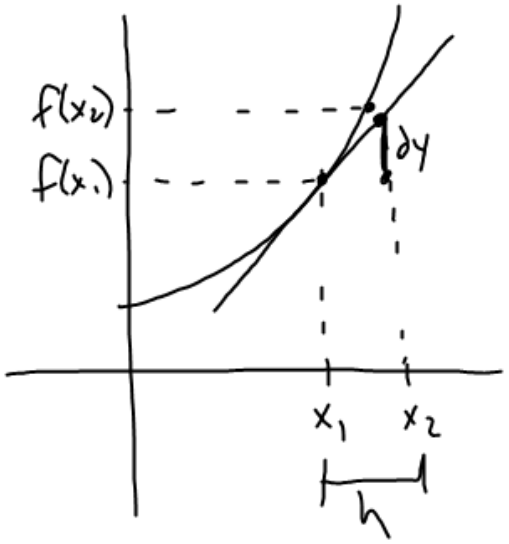
② Given $y=12x+5$, how much does y change when x changes by $2?\ 6?\ \frac{1}{3}?\ \frac{1}{12}?$ $=dx\cdot$ slope

$24\quad 72\quad 4\quad 1$ 24

Differential - infinitesimally small change in a variable

$dx = \Delta x = x_2 - x_1$

dy is the approximation (from $L(x)$) of $y_2 - y_1$

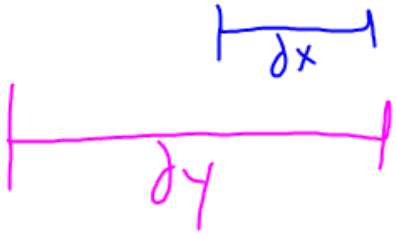


$\Longrightarrow dy = f'(x) \cdot dx$

$\frac{dy}{dx} = f'(x)$

$\frac{f(x+h) - f(x)}{h}$

$L(x) = f(a) + f'(a)(x - a)$



Find the differential dy and evaluate dy for given values of x and dx

① $y = x^5 + 37x$, $x = 1$ $dx = 0.01$

$dy = f'(x) \cdot dx$

$dy = (5x^4 + 37) \cdot dx$

$dy = (5(1)^4 + 37) \cdot (0.01)$

$dy = 0.42$

1

1.01

Find the change in volume of a sphere, $V = \frac{4}{3}\pi r^3$, when radius changes from 10 to 10.05cm.

$$20\pi \approx 62.832$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$dV = 4\pi r^2 \cdot dr$$

$$4\pi(10)^2 \cdot (0.05)$$

$$\frac{4}{3}\pi(10)^3 - \frac{4}{3}\pi(10.05)^3$$

$$\approx 63.14$$

#17 Newton's method $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ $L(x) = 0$

$x^2 - 2x + 1 = \sin x$

$0 = \sin x - x^2 + 2x - 1$

$x_{n+1} = 2 - \frac{f(2)}{f'(2)}$

Zeros at 0.38, 1.96

(7)

$f(x) = (1+x)^k \qquad a = 0$

$L(x) = f(a) + f'(a)(x-a)$

$1 + k(x)$

$f'(x) = k(1+x)^{k-1} \cdot (1)$

$f'(0) = k(1)^{k-1} \cdot 1$

$= k$

$f(0) = 1$

4.5

#20, 25, 26, 27, 31, 34 pick some from

35-40, 42, 44, 46, 47, 51